

The Role of Kinetic Energy Flux in the Convective Urca Process

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ABSTRACT

The previous analysis of the convective Urca neutrino loss process in degenerate, convective, quasi-static, carbon-burning cores by Barkat & Wheeler omitted specific consideration of the role of the kinetic energy flux. The arguments of Barkat & Wheeler that steady-state composition gradients exist are correct, but chemical equilibrium does not result in net cooling. Barkat & Wheeler included a “work” term that effectively removed energy from the total energy budget that could only have come from the kinetic energy, which must remain positive. Consideration of the kinetic energy in the thermodynamics of the convective Urca process shows that the convective Urca neutrinos reduce the *rate* of increase of entropy that would otherwise be associated with the input of nuclear energy and slow down the convective current, but, unlike the “thermal” Urca process do not reduce the entropy or temperature.

Subject Headings: Physical processes: convection – hydrodynamics – nuclear reactions – Stars: interiors – supernovae

1. Introduction

The convective Urca process has been discussed for over two decades in the literature and is still not satisfactorily understood or resolved. It should be just a matter of proper bookkeeping of the thermodynamic variables, but the problem has proved complex and subtle enough that even the sign of the effect is still debated.

The essence of the convective Urca process was first worked out by Paczyński (1972) for which the convective circulation driven by carbon burning in degenerate white dwarfs will yield first electron capture and then β -decay of susceptible nuclei. This will yield no net change in composition, but a loss of neutrinos (or antineutrinos) at each step of the cycle along with their attendant energy. Paczyński thus argued that this cycle would catalyze an energy loss that would cool the star and postpone dynamical runaway. This was contested by Bruenn (1973) who pointed out that, microscopically, each weak interaction added heat to the system, despite the loss of the neutrino. As electrons were captured below the Fermi sea, another electron dropped from the Fermi surface to fill the “hole,” resulting in heat, or β -decay deposited electrons with excess thermal energy above the Fermi sea on the other half of the cycle. It is important to determine the effect of the convective Urca process because it has direct observational implications for Type Ia supernovae. If a carbon/oxygen white dwarf of the Chandrasekhar mass undergoes thermonuclear runaway at the density at which carbon ignites, then the density is relatively low and subsequent electron capture and neutronizing reactions in the explosion are minimized. If, on the other hand, the convective Urca process substantially postpones the density of dynamical runaway to higher values, the attendant neutronization in the thermonuclear combustion can lead to unacceptably high neutron enrichment of the ejecta.

The physics of the convective Urca process was most recently analyzed in detail by Barkat & Wheeler (1990; hereafter BW), who argued that convective currents of

composition play a critical role in the quasi-steady state thermodynamics of the convective Urca process, so that cooling terms associated with the convective flow exactly cancel the microscopic heating terms described by Bruenn and lead to net cooling. Their results seemed to be consistent with the results of the careful numerical work of Iben (1978a,b, 1982) including a potential understanding of the cooling instability of a convective core containing an Urca shell found by Iben. In BW the instability resulted from the interaction of the convective core with the surrounding regions cooled by standard, plasma neutrino processes.

The conclusions of BW were called into question by Mochkovitch (1996). Mochkovitch presented a general thermodynamic analysis and concluded that the convective Urca process must heat the star. The conclusions of Mochkovitch were presented in a somewhat misleading way by implying that a paper by Lazareff (1975) was correct in its analysis and conclusion that the rate of change of entropy, ds/dt , must be positive. The conclusion of Lazareff may be true, but his argument was based on a very formal criterion that can be violated in principle, as argued by BW. The conclusion of Mochkovitch may also be correct, but his analysis did not specifically reveal the flaw in the physical arguments of BW.

In this paper we examine the issue yet again. We conclude, in agreement with Mochkovitch, that while the neutrinos associated with the convective Urca process carry away energy, the entropy, and the temperature, cannot decline. Rather, the convective Urca neutrino losses slow the convective currents and reduce the rate of increase of entropy associated with the input of nuclear energy. In §2, we make the case for the crucial role of the kinetic energy carried by the convective current. In the appendix we argue that proper treatment of the role of kinetic energy was the physical component missing in the analysis of BW. BW included a “work” term that effectively removed energy from the total energy budget that could only have come from the kinetic energy, which must remain positive.

The loss rate by the Urca process cannot exceed the rate of generation of kinetic energy. The role of buoyancy and mixing in creating and limiting the kinetic energy are outlined in §3 and §4 presents our conclusions.

2. Kinetic Energy of the Convective Current

The average radial velocity of the matter in a white dwarf with central carbon burning is negligible in the quasi-static phase because for every upwelling current there is a downflow. This does not mean that the average speed of the convective current and the kinetic energy associated with it are negligible. The role of this kinetic motion can be substantial even if the instantaneous value of the kinetic energy of the flow is small in terms of the total energy budget. As will be shown in §3, a significant portion of the nuclear energy released is deposited in the form of a buoyancy potential energy that is subsequently converted into kinetic energy that is, in turn, dissipated into thermal energy and Urca losses. The convective kinetic energy current is a key ingredient in turning the nuclear energy input that is produced in a relatively confined volume of the core into thermal energy spread throughout the convective core.

In many considerations of convection, the kinetic energy flux is treated as part of the “convective luminosity,” because one does not care whether heat is carried as thermal or kinetic energy, as long as it is spread around evenly to keep the entropy nearly constant in regions of efficient convection. This picture is correct as long as the convective current is a passive carrier of energy and particles, i.e. as long as the current itself does not have to do actual work to move the material around. When the convective Urca process is operating, however, there is a net electron current from the (average) radius at which the electrons are deposited by β -decay, to the (average) radius at which they are recaptured. The convective current must carry these electrons “uphill,” from low chemical potential to high chemical

potential (Couch & Arnett 1975). This work is performed specifically at the expense of the kinetic energy of the convective flow, not, directly, the nuclear input. If there were no Urca neutrino losses, the kinetic energy of the current would ultimately be expended as heat as the kinetic energy is dissipated by turbulent mixing. With the Urca neutrino processes, some of the work done goes into the energy carried off by the neutrinos. This energy is then not available to heat the matter.

Picture a schematic version of the convective core in which the nuclear input is confined in a rather small volume. The nuclear input creates a superadiabatic temperature gradient that generates convection throughout a significantly larger volume. The convection is assumed to reach an Urca shell, where the Fermi energy is equal to the difference in rest mass of Mother and Daughter nuclei (plus an electron) so β -decay and neutrino loss will occur, converting Mothers to Daughters. Subsequently, convection will carry Daughters inward and they will be converted back to Mothers with the production and loss of an anti-neutrino. Consideration of the work done in this cycle and the kinetic energy of the convective flow puts limits on the amount of energy that can be carried out by the neutrinos:

- a. Only a fraction, f_1 , of the nuclear energy input ($q = q_{nuc} - q_\nu$) can be converted into kinetic energy.
- b. Because of the dissipation associated with turbulent mixing, only a fraction, f_2 , of the kinetic energy can reach the Urca shell. This fraction may depend on the nature of the mixing, e.g. eddies or plumes.
- c. Only a fraction, f_3 , of the work done by the kinetic energy in lifting the Urca nuclei against the gradient in Fermi energy is converted into neutrinos. The remainder (about 25 percent, Bruenn(1973)) is converted to heat, thus $f_3 \leq 3/4$.

It is then immediately clear that Urca neutrino losses *can not amount to more than* $\sim \frac{3}{4}q$, even if $f_1 = f_2 = 1$.

We shall show in the next section that in the quasi-steady-state f_1 can be reasonably well estimated. The fraction f_2 is hard to estimate.

We argue that the net effect of the convective Urca process is twofold:

1. It *reduces the rate of increase* of the entropy due to nuclear burning. The convective Urca process includes 3 components: β -decay/capture, mixing, and convection. The first two components increase the entropy and the third cannot decrease it. The convective Urca process thus cannot reduce the entropy in an absolute sense, and likewise cannot reduce the temperature (for constant or increasing density), only the rate of increase of temperature.

2. It slows down the speed of convection beyond the Urca shell. Under the usual assumption that convective timescales are rapid compared to those of the β -processes, the rate of capture and decay of Urca nuclei depends only on the extent of the convective zone on both sides of the Urca shell. When the Urca shell is not too close to the center of the star, which is typically the case, convection is stopped shortly beyond the Urca shell. The heat created by the Urca process near the Urca shell further tends to flatten the temperature gradient locally there and hence to reduce the buoyancy within the convective core and thus contributes to the slowing down of the convection. The restriction of the size of the convective core by the Urca process limits the entropy at the edge of the convective zone. Since the expansion of the convective core is restricted, the core is prevented from growing to the traditional limit where the (nearly constant) entropy of the inner convective core is equal to the entropy of the outer stable layers with positive entropy gradient. The likely result is that a boundary region of negative entropy gradient will be established between the fully convective core and the outer stable regions.

In the appendix we examine the role of the kinetic energy by examining the neglect of its explicit treatment by BW.

3. Buoyancy and Kinetic Energy.

The Urca loss rates are limited by the kinetic energy in the convective flow. If the losses were to become so strong that the currents were slowed, the losses would be limited. A limit to the kinetic energy in the convective flow is given by the rate of production of the buoyancy potential energy by the thermonuclear reactions. Here we estimate the maximum rate at which buoyancy potential can be generated, giving an upper limit to the rate of creation of the kinetic energy (i.e. f_1).

Consider a convective region at the center of the star, with constant entropy density gradient $\frac{\partial s}{\partial M}$. At the center of the convective zone there is a heat source q_{nuc} . There is no other source or sink in the convective zone. For simplicity, neglect conductive heat transport and the “normal” neutrino losses that would limit the convective core if there were no Urca process. We choose a time step dt , which is large compared to the convection turnaround time, but small enough, that $q_{nuc}dt$ is small. Let us assume for the moment that the matter outside the initial convective region does not move, and so all the heat produced is contained in this initial convective region. The heat created during the interval dt is:

$$dQ = \left(\int_0^{M_{cc}} q_{nuc} dM \right) dt, \quad (1)$$

where M_{cc} is the mass of the convective core. The associated total entropy change of the convective region is then:

$$dS = \left(\int_0^{M_{cc}} \frac{q_{nuc}}{T} dM \right) dt = 1/T_q dQ, \quad (2)$$

where $1/T_q$ is an average of $1/T$ over the region where $q_{nuc} > 0$:

$$1/T_q \equiv \left(\int_0^{M_{cc}} \frac{q_{nuc}}{T} dM \right) / \left(\int_0^{M_{cc}} q_{nuc} dM \right). \quad (3)$$

To the extent that q_{nuc} is highly centrally concentrated, T_q will be close to the central temperature of the core.

Within the convective region, the mass that is heated by the nuclear reactions at the center of the region is assumed to move adiabatically to rapidly mix its heat smoothly throughout the convective region by sharing its entropy with the larger mass of the whole convective region. The resulting change in the specific entropy, $ds = dS/M_{cc}$, is then independent of the position within the convective core. We could view this process as comprising two steps: coarse mixing by circulation that spreads the heated mass evenly over the convective zone without changing the entropy, then a fine local mixing, which changes the entropy. This change of entropy in the second step is a second-order effect and can be ignored. With these assumptions, the thermal energy of the convective zone is increased by:

$$dE = \int_0^{M_{cc}} (T ds) dM = \frac{dS}{M_{cc}} \int_0^{M_{cc}} T dM = dS T_{cc} = T_{cc}/T_q dQ, \quad (4)$$

where T_{cc} is an average of T over the convective zone:

$$T_{cc} \equiv (\int_0^{M_{cc}} T dM) / (\int_0^{M_{cc}} dM). \quad (5)$$

The buoyancy dB is the difference between the energy dQ released by the reaction and the final change dE in the thermal energy:

$$dB = dQ - dE = dQ(1 - T_{cc}/T_q), \quad (6)$$

or:

$$f_1 = 1 - T_{cc}/T_q \quad (7)$$

The value of f_1 can be computed straightforwardly for a convective core with no Urca processes. The presence of the Urca process that restricts the growth of the convective core will generate a region with negative entropy just beyond the Urca shell, as mentioned in §2.

The effect of this boundary gradient on the fraction of the input energy that is converted to kinetic energy is difficult to estimate. The effect of this region of negative entropy gradient is being explored with 2D numerical simulations.

4. Conclusions

We conclude here: a) In agreement with Mochkovitch (1996) and in disagreement with the earlier work in BW, that the convective Urca process can reduce the rate of heating by nuclear reactions, but cannot result in a net decrease in entropy, and hence in temperature for a constant or increasing density; b) The convective Urca process must limit the expansion of the convective zone beyond the Urca shell. The error in BW is traced back to ignoring the role of the kinetic energy (See appendix).

An interesting precursor to these conclusions is the work of Finzi & Wolf (1968) on the pulsational Urca process. Finzi & Wolf show that a system makes a transition from a “thermal Urca” regime to a “vibrational Urca” regime when the vibrational energy becomes large compared to kT . The rate of dissipation of the vibrational energy grows with the vibrational energy with part of the dissipated energy going into neutrino losses and the remainder into thermal energy. The dissipation of the vibrational energy thus drives the pulsational Urca neutrino losses. The neutrino losses never, of course, dissipate vibrational energy faster than it is being pumped in, and the vibrational energy can only be reduced to zero without cutting off the pulsational neutrino loss process. In our problem, the convective Urca losses grow as the convective core extends further beyond the Urca shell. The limit established above on Urca losses thus prevents the convection from extending far beyond the Urca shell.

While we agree with the general conclusions of Mochkovitch (1996), we feel that his

simple picture of the convective core should, perhaps, be taken with a note of caution. He follows the argument of Couch & Arnett (1975) to write the work done by convection as $w = \epsilon_F^{ec} - \epsilon_F^{em}$ and then uses this equality to establish some interesting inequalities that suggest that convective Urca losses cannot stabilize nuclear burning and that the convective heat engine needs a cold source, the material at the boundary of convective core. These conclusions may be true, but the work done by convection must not only account for the difference in the Fermi energy at electron capture and emission, but also the energy loss by dissipation of the convective currents and thus $w > \epsilon_F^{ec} - \epsilon_F^{em}$.

Over the last couple of decades most work on modeling of the progenitors of Type Ia supernovae has simply ignored the effects of the convective Urca process. The conclusion we have reached, after similar decades of pondering, is that that was probably the right approach.

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Appendix: The Barkat - Wheeler Paper

BW do not refer directly to the convective kinetic energy. They define the convective luminosity profile $L_{cc}(M, t)$ which is assumed to adjust instantaneously to the slowly varying nuclear heat source, q . The equations of §2 of BW are consistent with the interpretation that the kinetic energy is included in the “internal energy” carried by the convective flux and hence that L_{cc} represents the flux of both the thermal and kinetic energy (note that

the small contribution from the diffusion of energy is ignored). The kinetic energy can be created at a high rate by the nuclear reactions and yet destroyed at a high rate by turbulent dissipation. The kinetic energy can hence be small at any given instant in the quasi-static approximation and almost constant in time.

In addition to its conceptual and physical role in the conversion of nuclear to thermal and neutrino energy, it is important to keep track of the kinetic energy separately even though it may be small, because *the kinetic energy must be positive*, and it is the kinetic energy that drives the Urca process and that is converted into neutrinos and heat by the Urca process. This allows an upper limit to be placed on the Urca losses. The Urca losses can only remove energy at a rate that does not reduce the kinetic energy to zero, otherwise the convective Urca process will shut down.

Here we will argue that the formulation of BW was essentially correct with the interpretation that the internal energy includes the kinetic energy. The problem arises in BW because the Urca losses come from the kinetic energy and the kinetic energy is positive. This effectively restricts the Urca losses. If the kinetic energy is treated as an indistinguishable part of the total internal “thermal” energy, then this restriction is not manifest.

In particular, BW correctly argue that the Urca convective core must be close to chemical equilibrium with gradients in the composition of the Urca-active components that lead to steady-state currents. BW err in the manner in which they invoke chemical equilibrium in the entropy equation. BW include a term that implies that the current of electrons does work against the full electron chemical potential gradient and subtract that work, the critical “cooling term,” from the “heating terms” in the entropy equation. We stress the crucial point that this subtraction was done in the entropy equation, not in the energy equation. The treatment in BW in essence says that the current of electrons brings

electrons with a *negative* entropy equivalent to $-|\epsilon_F - \epsilon_{th}|/T$ into the Urca shell (and carries out electrons with *positive* entropy $|\epsilon_{th} - \epsilon_F|/T$ on the other side of the Urca shell), where ϵ_F is electron Fermi energy, and ϵ_{th} is the threshold energy for electron capture. BW added that negative entropy to the entropy in a given mass element. Instead, the entropy brought in by the current of Urca-active elements is small, only that represented by the thermal component of the electron energy, and it is positive. Since $\epsilon_F - \epsilon_{th} \gg kT$ except directly at the Urca shell, the effect of the term invoked by BW is to drastically exaggerate the loss of entropy.

BW correctly write the expression for the loss of entropy due to neutrino processes in the absence of convection in their equation 8:

$$T \frac{ds}{dt} = q_{nuc} - q_\nu - \frac{\partial L_{diff}}{\partial M} - \epsilon_\nu \left| \frac{dN_e}{dt} \right| - \sum_i \mu_i \frac{dN_i}{dt}, \quad (1)$$

where q_{nuc} is the nuclear input, q_ν is the standard (plasmon) neutrino loss, $\frac{\partial L_{diff}}{\partial M}$ is the (negligible) energy carried by radiation diffusion, and $\epsilon_\nu \left| \frac{dN_e}{dt} \right|$ represents the non-convective contribution of the thermal Urca processes. Since $\sum_i \mu_i \frac{dN_i}{dt} = -|(\epsilon_F - \epsilon_{th})| \left| \frac{dN_e}{dt} \right|$, Eq. 1 can be written as:

$$T \frac{ds}{dt} = q_{nuc} - q_\nu - \frac{\partial L_{diff}}{\partial M} + (|\epsilon_F - \epsilon_{th}| - \epsilon_\nu) \left| \frac{dN_e}{dt} \right|, \quad (2)$$

where the last term summarizes the net Urca heating. BW define $\epsilon_+ = |\epsilon_F - \epsilon_{th}|$ and $\epsilon_h = \epsilon_+ - \epsilon_\nu$) such that the final heating term in Eq. 2 is $\epsilon_h |dN_e/dt|$.

The correct expression for the entropy in the presence of a convective flow and associated neutrino losses is:

$$T \frac{\partial s}{\partial t} = T \frac{ds}{dt} - \frac{\partial L_{cc}}{\partial M}, \quad (3)$$

where the local contribution to the change in entropy Tds/dt is given by Eq. 2, and the second term on the right hand side represents the contribution due to the currents, i.e., the convective luminosity. The details of how one should write the convective luminosity are

difficult to define and implement in practice and may not be important, given our basic conclusion that, at best, the Urca process can only limit the rate of growth of entropy.

In their attempt to include the effects of Urca convection on the entropy balance, BW write their equation 29:

$$T \frac{\partial s}{\partial t} = q_{nuc} - q_\nu - \frac{\partial L_{diff}}{\partial M} - \epsilon_\nu \left| \frac{dN_e}{dt} \right| - \frac{\partial L_{cc}}{\partial M} - \sum_i \mu_i \frac{\partial N_i}{\partial t}. \quad (4)$$

Note that there is a typo in BW equation 29, the entropy derivative is with respect to time, not temperature, and that the term q is not clearly defined. The term “ q ” as used in BW Eq. 29 is not the q of BW equation 5, but rather the first four terms of Eq. 4 as we have written them explicitly here. The error in BW is in the term $-\sum_i \mu_i \partial N_i / \partial t$. BW break this sum into two terms, the local Lagrangian term $-\sum_i \mu_i dN_i / dt$ as included in Eq. 2 here, and a term due to the current of electrons, $\sum_i \mu_i \partial L_{N_i} / \partial M$. The problem is with the latter term. The current of electrons and Urca-active nuclei cannot carry away the implied quantity of entropy.

The meaning of the term $\sum_i \mu_i \partial L_{N_i} / \partial M$ becomes more clear when we integrate it over the whole convective region. Integrating by parts, and remembering that L_{N_i} vanishes on the boundaries, we get:

$$-\int_0^{M_{cc}} \sum_i L_{N_i} \partial \mu_i / \partial M = -\int_0^{M_{cc}} \sum_i L_e \partial \epsilon_f / \partial M, \quad (5)$$

where we have neglected the chemical potential of the ions. The absolute value of the last expression is the work done by the current in pushing the extra electrons “uphill.” Therefore the expression in Eq. 5 represents the loss of kinetic energy associated with this work and not the change in entropy. This term should *not* have been included in the entropy equation Eq. 29 of BW. For illustration, we have included the same erroneous term here in Eq. 4, but it is incorrect for the same reason. BW Eq. 30 is formally correct, representing the translation from Lagrangian to Eulerian coordinates, but not applicable to Eq. 29, since

the last term in that equation and in Eq. 4 as written here should be the Lagrangian component only (as given here in Eq. 1), not the Eulerian component.

BW employ the argument that by chemical equilibrium a suitable average over the convective core must give $\sum_i \mu_i \frac{\partial N_i}{\partial t} = 0$. They thus effectively argue that the last term in Eq. 4 (or their Eq. 29) is zero and that the remaining terms result in net cooling. To see this in a bit more detail, this expression can be decomposed as $\sum_i \mu_i \frac{\partial N_i}{\partial t} = \sum_i \mu_i \frac{dN_i}{dt} - \sum_i \mu_i \frac{\partial L_{N_i}}{\partial M} = 0$. Note from the discussion following Eq. 1 that $\sum_i \mu_i \frac{dN_i}{dt} = -\epsilon_+ \left| \frac{dN_e}{dt} \right|$. The application of chemical equilibrium thus also gives $\sum_i \mu_i \frac{\partial L_{N_i}}{\partial M} = -\epsilon_+ \left| \frac{dN_e}{dt} \right|$. The latter term is thus a putative cooling term due to currents that offsets the heating term due to local Urca heating. Since this “cooling” term should not be present in Eq. 4, we conclude that chemical equilibrium is a reasonable assumption for this system, but that it does not play any direct role in the analysis of the thermodynamics of the convective Urca process. Given the erroneous introduction of the “cooling” term in their Eq. 29, the subsequent discussion in BW (through Eq. 37 ff) is irrelevant. We note that Eq. 37 as presented in BW, even if corrected, is not a good way to represent the growth of entropy in time. The analog would be an integral of Eq. 1 over the mass of the convective core, but such a procedure would neglect the strong feedback of the Urca process on the convective current. Inclusion of the term in Eq. 5 in the energy equation, as done by Iben (1978a; Eq. 8), is allowed. Care must be taken, however, to include its affects only on the kinetic energy, which must always be non-negative.

Much of the past work on the convective Urca process has assumed that the size of the convective core is defined by the condition that the convective Urca losses (plus minor standard neutrino losses) be equal to the rate of nuclear input. Since the convective Urca process cannot actually reduce the entropy of the convective core, this definition of the convective core is invalid. The size of the convective core must instead be set by the normal

processes of the entropy profile plus some overshoot. The convective Urca process may affect the size of the core by slowing the speed of convective currents when the boundary of the core exceeds the radius of an Urca shell. More specifically, we note that the core stability analysis of BW is invalid because it depended specifically on equating the Urca losses with nuclear input.

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